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$-(h/b)$, and the tangent of the angle (10) makes with the y axis is $\tan\theta = -(b/h) \dots (11)$. Substituting $a = a - \lambda$ in (4), we find

$$u = \frac{1}{e^2} = \frac{a - \lambda + b}{\sqrt{[(a - \lambda - b)^2 + 4h^2]}} + \frac{1}{2} \dots (12).$$

Equating $du/d\lambda$ to zero, we find $(a - \lambda)b - b^2 - 2h^2 = 0$, or,

$$a - \lambda = \frac{b^2 + 2h^2}{b} \dots (13). \quad \text{Then } a - \lambda - b = \frac{2h^2}{b} \dots (14).$$

Substituting (7), (13) and (14) in (4) and reducing

$$e^4 + \frac{4h^2}{b^2}(e^2 - 1) = 0, \text{ or } \frac{b^2}{h^2}e^4 + 4e^2 - 4 = 0 \dots (15).$$

This becomes by (11), $e^4 \tan^2 \varphi + 4e^2 - 4 = 0 \dots (16)$.

Also solved by G. W. GREENWOOD, B. A. (Oxon), Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill., and by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

171. Proposed by J. E. SANDERS, Hackney, Ohio.

A thread passes spirally around a rough cylinder 10 feet high and 6 inches in diameter. How far will a pigeon fly in unwinding the thread if the distance between the coils is 4 inches, and the thread unwound is at all times horizontal?

I. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, West Va.

Let r = radius of cylinder = 3 inches = $\frac{1}{4}$ feet; d = distance between coils = $\frac{1}{3}$ feet; m = number of coils = 30. Then from Vol. I, No. 6, pages 318-320 of this Journal* we have for the required distance

$$S = \pi m^2 \sqrt{(4\pi^2 r^2 + d^2)} \text{ feet} = 900\pi \sqrt{(\frac{1}{4}\pi^2 + \frac{1}{9})} \text{ feet} = 4540.246 \text{ feet, nearly.}$$

II. Solution by H. B. LEONARD, B. S.

Circumference = 6π . Length of thread on one turn = $\sqrt{(6\pi^2 + 4^2)}$. Number of turns = $(12 \times 10) \div 4 = 30$. Total rotation = 60π . Angle of elevation of thread on cylinder = $a = \sin^{-1} \frac{4}{\sqrt{(6\pi^2 + 4^2)}}$. During the bird's flight, an unwinding of $d\theta$ produces an increase of $dz = 3 \tan a \cdot d\theta$ in altitude, of $dr = 3 \sec a \cdot d\theta$ in direction of flight, of $dc = 3\sqrt{(\theta^2 \sec^2 a + 1)} d\theta$ normal to direction of flight.

$$\begin{aligned} (ds)^2 &= (dr)^2 + (dz)^2 + (dc)^2 = (9 \sec^2 a + 9 \tan^2 a + 9 \theta^2 \sec^2 a + 9) (d\theta)^2 \\ &= (18 \sec^2 a + 9 \theta^2 \sec^2 a) (d\theta)^2. \quad ds = 3 \sec a \sqrt{(2 + \theta^2)} d\theta. \end{aligned}$$

$$S = \int_0^{60\pi} 3 \sec a \sqrt{(2 + \theta^2)} d\theta = 3 \sec a \int_0^{60\pi} \sqrt{(2 + \theta^2)} d\theta = 3 \sec a \left[\frac{1}{2} \{ \theta \sqrt{(2 + \theta^2)} \right.$$

* See also Vol. I, pp. 88-89. Ed.

$$\begin{aligned}
& + 2\log[\theta + \sqrt{(2 + \theta^2)}] \Big]_{\theta=0}^{\theta=60} = 3\sec\alpha \{ \frac{1}{2} [60\pi\sqrt{(2 + 3600\pi^2)} \\
& \quad + 2\log[60\pi + \sqrt{(2 + 3600\pi^2)}] - \frac{1}{2} [0\sqrt{2} + 2\log(0 + \sqrt{2})] \} \\
& = 3\sec\alpha \{ 30\pi\sqrt{(2 + 3600\pi^2)} + \log[60\pi + \sqrt{(2 + 3600\pi^2)}] - \log\sqrt{2} \} = 4540 \text{ feet.}
\end{aligned}$$

DIOPHANTINE ANALYSIS.

116. Proposed by HARRY S. VANDIVER, Bala, Pa.

If n is an odd positive integer, and $1, n, n', n'', \dots$ denote all its distinct divisors, then $2^n > 2[n+1][n'+1][n''+1]\dots$

Solution by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

There is a single exception $n=3$, for which $2^3=2[3+1]$. The corrected theorem may be proved by induction, using the following lemma:

If p is an odd prime number and d and π positive integers,

$$[1+d]^{p^\pi} > [1+d][1+pd][1+p^2d]\dots[1+p^\pi d],$$

except for $d=1, \pi=1, p=3$, the equality sign then holding.

For proof we apply $p^\pi - 1 = p - 1 + p[p-1] + p^2[p-1] + \dots + p^{\pi-1}[p-1]$.

$$\therefore [1+d]^{p^\pi} = [1+d][1+d]^{p-1}[1+d]^{p(p-1)}\dots[1+d]^{p^{\pi-1}(p-1)}.$$

But $[1+d]^{t(p-1)} \geq [1+td]^{p-1} \geq 1 + [p-1]td + [td]^{p-1} \geq 1 + [p-1]td + td$, if $p > 2$. The equality signs hold simultaneously only when $t=d=1, p=3$. Hence, for $v > 2$, $[1+d]^{t(p-1)} > 1 + ptd$ unless $t=d=1, p=3$, so that the lemma follows.

To prove the theorem by induction, we note that it is true for $n=p^\pi > 3$, in view of the lemma for $d=1$. Assume that it has been verified for $n=p_1^{\pi_1} p_2^{\pi_2} \dots$. We proceed to prove it true for $N=np^\pi$, p being prime to n . We have

$$2^N = [2^n]^{p^\pi} \geq [(1+1)(1+n)(1+n)\dots]^{p^\pi}$$

$$\geq [(1+1)(1+p)\dots(1+p^\pi)][(1+n)(1+pn)(1+p^2n)\dots][(1+n')(1+p'n')\dots],$$

in view of the lemma. But the distinct divisors of N are

$$1, n, n', \dots, p, pn, pn', \dots, p^2, p^2n, \dots, p^\pi, p^\pi n, \dots$$

The theorem is therefore true for N .

118. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Find the two least integral numbers such that their sum shall be a square and the sum of their squares a biquadrate.

Solution by G. B. M. ZERE, A. M., Ph. D., Parsons, W. Va.

Let x and y be the numbers, then for $x+y=1, x^2+y^2=13^4$,
 $x=120, y=-119$.